

MAT 221

INTRODUCTION TO LINEAR ALGEBRA

3 cr. (3-0)

**COURSE DESCRIPTION:**

MAT 221 is an introduction to the theory and application of linear algebra. Topics include systems of linear equations, matrices, determinants, vector spaces, inner product spaces, linear transformations, and the eigenvalue problem. Emphasis is placed on the application of linear algebra and formal verification of theoretical properties. Applications include polynomial curve fitting, network analysis, stochastic matrices, Leontief Input-Output models, least squares regression analysis, eigenvalue problems, applications in analytic geometry, and least squares approximations. The Texas Instruments TI-83 or TI-84 graphing calculator or a calculator approved by the instructor is required for this course. **MAT 221 is ordinarily offered in the spring semester in even numbered years.**

**PREREQUISITES:**

MAT 201 with a grade of “C” or higher

**COURSE OBJECTIVES:**

1. Write a parametric representation for a solution set to a system of linear equations.
2. Apply back-substitution to solve a system of linear equations.
3. Determine whether a system of linear equations is consistent or inconsistent.
4. Utilize an augmented matrix and elementary row operations to solve a system of linear equations.
5. Determine whether a matrix is in row-echelon form or reduced row-echelon form.
6. Apply Gaussian elimination to solve a system of linear equations.
7. Apply Gauss-Jordan elimination to solve a system of linear equations.
8. Perform matrix operations including addition, subtraction, scalar multiplication, matrix multiplication, and transposition.
9. Compute the inverse of a matrix (if it exists) as well as the inverse of a matrix product (if it exists).
10. Factor a matrix into a product of elementary matrices.
11. Calculate the LU-factorization of a matrix and use it to solve an associated system of linear equations.
12. Utilize minors and cofactors to calculate the determinant of a square matrix.
13. Use elementary row or column operations to evaluate the determinant of a square matrix.
14. Identify conditions that yield zero determinants.
15. Utilize the determinant to decide whether a square matrix is singular or non-singular.

16. Apply properties of determinants relating to matrix products, scalar products, inverses, and transposition.
17. Calculate and verify eigenvalues and eigenvectors of second and third-order square matrices.
18. Calculate and use the adjoint of a square matrix to find its inverse.
19. Utilize Cramer's Rule to solve a system of linear equations.
20. Use a determinant to calculate the area of a triangle, test for point collinearity, find the equation of a line, calculate the volume of a tetrahedron, test for point coplanarity, and find the equation of a plane.
21. Utilize properties of vector operations to perform calculations on vectors in real  $n$ -space.
22. Determine if a set of vectors along with two defined operations forms a vector space.
23. Determine if a subset of a vector space forms a subspace.
24. Write a vector as a linear combination of a finite set of vectors (if possible).
25. Determine if a set of vectors from a vector space forms a spanning set for that space.
26. Determine the linear independence or dependence of a finite set of vectors from a vector space.
27. Determine if a finite set of vectors forms a basis for a vector space.
28. Find a basis for and the dimension of the column space, row space, and nullspace (nullity) of a matrix.
29. Find dimensions of vector spaces and subspaces.
30. Calculate a coordinate matrix of a vector relative to a given basis of a vector space as well as a transition matrix and perform a change to a new basis within that space.
31. Determine if a function defines an inner product on a vector space.
32. Utilize Euclidean and non-Euclidean inner products to determine vector lengths, unit vectors, distance between vectors, angles between vectors, vector orthogonality, vector parallelism, vector orthonormality, vector projections, as well as actual results of vector inner products.
33. Calculate the cross product of two vectors from real 3-space.
34. Apply the Gram-Schmidt orthonormalization process to find an orthonormal basis for a given basis, subspace, or inner product space.
35. Determine whether subspaces are orthogonal.
36. Find the orthogonal complement of a subspace.
37. Utilize a set of normal equations to find the least squares solution of an associated system.
38. For two given vectors, verify the Cauchy-Schwartz Inequality, Triangle Inequality, and Pythagorean Theorem.
39. Find the image and preimage of a function.
40. Determine whether a function from one vector space to another is a linear transformation.
41. Find the kernel and range of a linear transformation as well as a basis for each.
42. Determine if a linear transformation is one-to-one or onto.
43. Calculate the nullity and rank of a linear transformation.

44. Verify that a given matrix defines a linear function that is one-to-one and onto.
45. Determine whether two vector spaces are isomorphic.
46. Find the standard matrix for a given linear transformation and use it to compute the image of a given vector.
47. Find the standard matrix of the composition of linear transformations.
48. Calculate the inverse of a linear transformation (if it exists).
49. Construct the matrix of a given linear transformation relative to a nonstandard basis.

## **COURSE OUTLINE:**

- I. SYSTEMS OF LINEAR EQUATIONS
  - A. Introduction to Systems of Linear Equations
  - B. Gaussian Elimination and Gauss-Jordan Elimination
  - C. Applications of Systems of Linear Equations
- II. MATRICES
  - A. Operations with Matrices
  - B. Properties of Matrix Operations
  - C. The Inverse of a Matrix
  - D. Elementary Matrices
  - E. Applications of Matrix Operations
- III. DETERMINANTS
  - A. The Determinant of a Matrix
  - B. Evaluation of a Determinant Using Elementary Operations
  - C. Properties of Determinants
  - D. Introduction to Eigenvalues
  - E. Applications of Determinants
- IV. VECTOR SPACES
  - A. Vectors in  $\mathbb{R}^n$
  - B. Vector Spaces
  - C. Subspaces of Vector Spaces
  - D. Spanning Sets and Linear Independence
  - E. Basis and Dimension
  - F. Rank of a Matrix and Systems of Linear Equations
  - G. Coordinates and Change of Basis
  - H. Applications of Vector Spaces
- V. INNER PRODUCT SPACES
  - A. Length and Dot Product in  $\mathbb{R}^n$
  - B. Inner Product Spaces
  - C. Orthonormal Bases: Gram-Schmidt Process
  - D. Mathematical Models and Least Squares Analysis
  - E. Applications of Inner Product Spaces

## VI. LINEAR TRANSFORMATIONS

- A. Introduction to Linear Transformations
- B. The Kernel and Range of a Linear Transformation
- C. Matrices for Linear Transformations
- D. Transition Matrices and Similarity
- E. Applications of Linear Transformations

### COURSE REQUIREMENTS:

1. Homework. Homework will be assigned at every class session. The student should realize that, as a general rule of thumb, a minimum of two hours of study outside of class is required for every one hour of class time. This course will require a minimum of ten to fifteen hours per week of outside class work.
2. Attendance. Attendance in this class is both expected and required. John A. Logan College's attendance policy will be enforced:
  - a. Students are expected to attend all scheduled class periods for the courses in which they are enrolled unless they have been called for military duty, jury duty, or subpoenaed as a witness during regular school days, or are participating in a scheduled, supervised college trip or function. There are no excused absences or minimum number of class "cuts." All absences must be made up in a manner acceptable to the instructor.
  - b. A student who is absent from class for three consecutive meetings or who is excessively absent as defined by the instructor, without prior approval, may be required by the instructor to meet with the department chair or dean for instruction before being readmitted to the class. Students who claim illness as a cause for excessive absences may be required to present a physician's statement before being readmitted to the class.
  - c. Students should notify the dean of student services when extensive absences are necessary (due to illness, hospitalization, or a death in the family).
  - d. Students will be allowed to make up work for classes missed while on a scheduled, supervised College trip or function, a death in the immediate family, or for classes missed while serving on jury duty, or for serving as a witness in court. Instructors must be notified in person by the student not later than one class meeting prior to the absence. Students who have been summoned for jury duty must present a copy of the official notification or the subpoena to the instructor prior to the absence.
3. Required Materials. The textbook, along with the usual notebook, paper, pencils, etc., represent the required materials for the class. **Calculators (subject to the instructor's approval)** will be allowed on all tests. Also, in order to access supplementary materials for this course, students are required to have a **valid email account address**.

4. Online Supplementary Materials. A web-based program called Moodle will be used to access online supplementary class materials. For information on Moodle or logging into the online component of this course, please go to: <http://online.jalc.edu/>. Information about free online orientation to Moodle and online instruction is also available at the same location. Please note that one-half hour of **free college credit** can be earned for first-time completion of Moodle orientation. For more information, go to: <http://online.jalc.edu/>.

Student Success Center. Tutors may be obtained through the Student Success Center. Contact the staff in C219 if this service is desired. John A. Logan College will make reasonable accommodations for students with documented disabilities under Section 504 of the Rehabilitation Act of 1973, and the Americans with Disabilities Act of 1990. Any student with a disability that may have some impact on work in this class, who feels she/he needs an accommodation, should make an appointment with the Coordinator of Services for Students with Disabilities on campus, Jennifer Frost, Room C219B, Ext. 8516. Before services can be provided, this advisor must determine eligibility and arrange appropriate academic adjustments. ***It is the student's responsibility to register in advance of a school term with this office and to turn in a schedule each term to ensure that there is every opportunity for success in this class.***

English Writing Center/Tutoring. For assistance with writing assignments in any college course, students are encouraged to visit "The Write Place" in E109. English instructors are available for one-on-one tutoring each semester during hours posted at the center.

Financial Aid. Students that receive financial assistance and completely withdraw from classes prior to 60% of the semester being completed (approximately 2-3 weeks after midterm) could be responsible to return a portion of their Federal Pell Grant award. Prior to withdrawing from courses, students should contact the Financial Aid Office.

Course Withdrawal Information. It is expected that you will attend this class regularly. If you stop attending for any reason, you should contact your advisor and withdraw officially to avoid the posting of a failing grade (an E) to your transcript. It is also advisable to discuss the situation with your instructor before dropping.

### **METHOD OF EVALUATION:**

Evaluation will be made on the basis of:

1. A maximum of four 100-point exams that will be administered periodically over the course of the semester.
2. A 100-point final exam.

The combined average of the tests and final exam will determine the student's overall course percentage grade.

**No make-ups** will be given on the tests except for situations set forth in John A. Logan College's attendance policy. Any exam answers that have been copied from someone else will result in a zero for all parties involved.

Grades are assigned according to the following scale:

90 -	100% = A
80 -	89% = B
70 -	79% = C
60 -	69% = D
0 -	59% = E

### **METHOD OF PRESENTATION:**

Most instruction will be of the lecture-discussion type. Homework will be assigned which, together with announced exams, serve to amplify and clarify the classroom discussions. Occasionally, topics will be assigned to be performed through independent study. The student can obtain additional help through (1) free school tutoring in room C219, (2) the instructor during office hours, (3) the publisher's online resources.

### **TEXT:**

Elementary Linear Algebra; Sixth Edition, Larson and Falvo; Cengage, 2009.  
ISBN: 0-618-78376-8

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**DATE:** Spring, 2010

### **John A. Logan College Telephone Numbers**

Carterville and Williamson County.....	985-3741 (operator)
	985-2828 (direct extension access)
Carbondale and Jackson County .....	549-7335 (operator)
	457-7676 (direct extension access)
Du Quoin .....	542-8612
West Frankfort.....	937-3438
Crab Orchard, Gorham, & Trico areas .....	1-800-851-4720
TTY (hearing-impaired access) .....	985-2752

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